



ALMA MATER STUDIORUM  
UNIVERSITÀ DI BOLOGNA

# Type Systems for Distributed Programs: Components and Sessions

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## Gentle Intro: Features of Distributed Systems

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- Systems are designed as *structured composition of computational units* called **components**.
- Giving rise to **Component-Based Ubiquitous Systems** (CBUS)

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When reasoning about complex distributed systems, *reliability* and *usability* are of paramount importance.

- 1 **Reliability**: Systems need to account for **safe dynamic reconfiguration**, namely changing at runtime the communication patterns.
- 2 **Usability**: Components perform **communication** among each-other, following predefined *patterns* or *protocols*.

## Problem Description

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- ① Guaranteeing **consistency** of dynamic reconfigurations is a challenging task. It is difficult to ensure that modifications will not disrupt ongoing communications.
- ② Guaranteeing **safety** of communications means a collection of several requirements.
  - *privacy*
  - *communication safety*
  - *deadlock-freedom*
  - *livelock-freedom*

## Aim of the Ph.D. Dissertation

*To develop powerful techniques based on formal methods for the verification of correctness, consistency and safety properties related to dynamic reconfigurations and communications in complex distributed systems.*

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Static analysis based on **Types** and **Type Systems**. Why?

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  - concurrent programming: types for processes in the  $\pi$ -calculus
  - guarantee deadlock-freedom, livelock-freedom.

# Approach

Static analysis based on **Types** and **Type Systems**. Why?

- ① Types and Type Systems for **safety properties**.
  - concurrent programming: types for processes in the  $\pi$ -calculus
  - guarantee deadlock-freedom, livelock-freedom.
- ② Types and Type Systems for **communication**.
  - ranging from *standard channel types* to *behavioural types*, like **session types**.
  - guarantee privacy, communication safety, session fidelity.



# Contribution of the Ph.D. Dissertation

- i) We design a **type system** for a **component-based calculus**, to *statically* ensure consistency of dynamic reconfigurations.
- ii) We define an **encoding** of the  **$\pi$ -calculus with session types** into the **standard typed  $\pi$ -calculus**, to understand the expressive power of session types.
- iii) We relate the notions of **deadlock-freedom**, **livelock-freedom**, **progress** defined in **different calculi** via the encoding.

# Importance of the Contribution

## i) Type System for Components:

- 1 Guarantees **safe** dynamic reconfiguration.
- 2 Shifts checks from *runtime* to *compile time*.

## ii) Encoding of Session $\pi$ -calculus:

- 1 Reusability of existing theory of the standard typed  $\pi$ -calculus.
- 2 Robustness by subtyping, polymorphism, HO and recursion.
- 3 Expressivity result for session types: not many results on types.

## iii) Progress by Encoding:

- 1 Gives a systematic way of understanding the notions of **deadlock-freedom**, **livelock-freedom**, **progress**.
- 2 Encoding relates notions defined in different calculi.
- 3 Gives a new technique for guaranteeing progress.
- 4 More accurate analysis for progress property.

## Publications

- i) *Session Types Revisited*. O. Dardha, E. Giachino and D. Sangiorgi. In Proc. of [PPDP'12](#), pp 139–150, ACM, 2012.
- ii) *A Type System for Components*. O. Dardha, E. Giachino and M. Lienhardt. In Proc. of [SEFM'13](#), pp 167–181, Springer LNCS, 2013.
- iii) *Progress as Compositional Lock-Freedom*. M. Carbone, O. Dardha and F. Montesi. To appear in [COORDINATION'14](#), Springer LNCS, 2014.

## In the remainder...

- Safe Dynamic Reconfiguration
- Safe Communication by Encoding
- Progress of Communication

# Safe Dynamic Reconfiguration

# Component-Based Calculus

- **Asynchronous Object Communication**
  - Asynchronous method calls:  $x = o!m(args)$
  - Primitives to test and fetch the returned value.
- **Concurrent Object Groups - cog**
  - Cooperating Objects sharing the processor; only one task active at time.
  - A group's activity consists of a set of tasks, created by asynchronous method calls on objects of the group;
  - `new cog C()` creates a new object in a new group.
- **Dynamic Reconfiguration**
  - `rebind o.p = o'` operation of `ports` of objects.

# Component-Based Calculus in Practice: Clients, Server and Controller

```
...  
Client c1 = new Client (s);  
Client c2 = new cog Client (s);  
Ctrl c = new Ctrl(c1,c2)!updateServer(snew);
```

```
...  
Unit updateServer(Server snew) {  
    rebind c1.S = snew;  
    rebind c2.S = snew;  
}
```

# A Type System for Components

- Goal of the Type System

- ① check `rebind` performed internally to a cog.
- ② check `synchronous method call` performed internally to a cog.

- How do we do it?

Statically track cogs identity and membership to a cog.



# Component-Based Calculus in Practice: Clients, Server and Controller

```
...  
Client  $c_1$  = new Client (s);  $\leftarrow G$   
Client  $c_2$  = new cog Client (s);  $\leftarrow G'$   
Ctrl  $c$  = new Ctrl( $c_1, c_2$ )!updateServer( $s_{new}$ );  $\leftarrow G$ 
```

```
...  
Unit updateServer(Server  $s_{new}$ ) {  
    rebind  $c_1.S = s_{new}$  ;  
    rebind  $c_2.S = s_{new}$  ;  
}
```

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```

```
...  
Unit updateServer(Server  $s_{new}$ ) {  
    rebind  $c_1.S$  =  $s_{new}$ ; ok  
    rebind  $c_2.S$  =  $s_{new}$ ;  
}
```

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}
```

# Properties of the Type System for Safe Dynamic Reconfiguration

## Theorem (Main Result)

*Well-typed programs do not perform*

- i) *illegal **rebinding***
- ii) *illegal **synchronous method call***

# Safe Communication by Encoding

## Session Types in Practice: Equality Test

`server`  $\stackrel{\text{def}}{=} x?(nr1).x?(nr2).x!\langle nr1 == nr2 \rangle.0$

`client`  $\stackrel{\text{def}}{=} y!\langle 3 \rangle.y!\langle 5 \rangle.y?(eq).0$

The system is given by

$(\nu xy) (\text{server} \mid \text{client})$

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## Session Types vs. Standard $\pi$ -Types

- **Session types** are structured  $x : ?\text{Int}.\text{?Int}.\text{!Bool}.\text{end}$ ;
- **Standard  $\pi$ -channel types** specify the type of the carried value:  $x : \ell_i[\text{Int}]$  or  $x : \ell_o[\text{Int}]$ .
- **Encoding** is based on:
  - ① **Linearity** of  $\pi$ -calculus channel types;
  - ② **Input/Output** channel capabilities;
  - ③ **Continuation-Passing** principle.

# Encoding Session Types

Let

$$S = ?\text{Int}.\text{?Int}.\text{!Bool}.\text{end}$$

Then

$$\llbracket S \rrbracket = \ell_i[\text{Int}, \ell_i[\text{Int}, \ell_o[\text{Bool}, \emptyset[]]]]$$

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# Properties of the Encoding

## Theorem (On types)

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## Theorem (On reductions)

*Encoding preserves evaluation of programs.*

# Advanced Features on Safety by Encoding

Does the encoding handle extensions? Extend the calculi with:

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Does the encoding handle extensions? Extend the calculi with:

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**Theorems** ‘On types’ and ‘On reductions’ still hold.

# Progress of Communication

## Comparing Properties of Communication

- **Deadlock-Freedom**: communications eventually succeed, *unless the whole process diverges*. (Standard  $\pi$ )
- **Livelock-Freedom**: communications eventually succeed even if the whole process diverges. (Standard  $\pi$ )
- **Progress**: each *session*, once started, is guaranteed to satisfy all the requested interactions. (Session  $\pi$ )



# What can we say about Progress?

## Theorem

*Progress is a compositional form of livelock-freedom property.*

- We use the **encoding** to relate **progress** in the session  $\pi$ -calculus to **livelock-freedom** in the standard  $\pi$ -calculus.
- Reusability of type system and tools for livelock-freedom.
- **More accurate** analysis of the progress property.

## Progress in Practice: “Bad” Process

Consider

$$(\nu ab)(\nu cd)( a?(z).d!\langle z \rangle \mid c?(w).b!\langle w \rangle )$$

## Progress in Practice: “Bad” Process

Consider

$$(\nu ab)(\nu cd)( a?(z).d!\langle z \rangle \mid c?(w).b!\langle w \rangle )$$

By encoding we obtain the process:

$$(\nu x)(\nu y)( x?(z).y!\langle z \rangle \mid y?(w).x!\langle w \rangle )$$

The type system for livelock-freedom **rejects** it!

## Progress in Practice: “Good” Process

Consider the process

$$(\nu ab) \left( b! \langle 1 \rangle \mid (\nu cd) ( d! \langle 1 \rangle \mid c?(y).a?(z) ) \right)$$

## Progress in Practice: “Good” Process

Consider the process

$$(\nu ab) \left( b! \langle 1 \rangle \mid (\nu cd) ( d! \langle 1 \rangle \mid c?(y).a?(z) ) \right)$$

By the encoding we obtain the process:

$$(\nu k) \left( k! \langle 1 \rangle \mid (\nu t) ( t! \langle 1 \rangle \mid t?(y).k?(z) ) \right)$$

The type system for livelock-freedom **accepts** it!

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- **Approach:** **types** and **type systems**.
  - i) **Type system** for **safe dynamic reconfiguration** in a concurrent object-oriented language for distributed systems.
  - ii) **Encoding** of session  $\pi$ -calculus into standard typed  $\pi$ -calculus permitting **large reusability** of existing **theory** and **properties**.

## Conclusions and Future Work 1/2

- **Problem:** guaranteeing **consistency** and **safety** properties in distributed programs.
- **Approach:** **types** and **type systems**.
  - i) **Type system** for **safe dynamic reconfiguration** in a concurrent object-oriented language for distributed systems.
  - ii) **Encoding** of session  $\pi$ -calculus into standard typed  $\pi$ -calculus permitting **large reusability** of existing **theory** and **properties**.
  - iii) **Progress** in session  $\pi$ -calculus as **livelock-freedom** in standard typed  $\pi$ -calculus via **encoding**.

## Conclusions and Future Work 2/2

- **Type System for Components** relevant in practice: designed for component-extension of ABS used in HATS and Envisage.
- **Encoding of Session Types** relevant for BETTY and ABCD.
- Extend the encoding to more general settings than dyadic session types, in particular multiparty session types.
- Session Types in Practice (ABCD)
- **Tool** for progress property in session types. Progress in more general settings.

Thank You!!

## References I

During my PhD I produced the following 4 papers. The last one [4] is not part of my PhD thesis, as it resulted from my Master's work.



Marco Carbone, Ornela Dardha, and Fabrizio Montesi.

Progress as compositional lock-freedom.

In *COORDINATION*, volume 8459 of *LNCS*, pages 49–64.

Springer, 2014.



Ornela Dardha, Elena Giachino, and Michael Lienhardt.

A Type System for Components.

In *Software Engineering and Formal Methods - 11th*

*International Conference, SEFM*, volume 8137 of *LNCS*, pages

167–181. Springer, 2013.

## References II



Ornela Dardha, Elena Giachino, and Davide Sangiorgi.  
Session types revisited.

In *PPDP*, pages 139–150, New York, NY, USA, 2012. ACM.



Ornela Dardha, Daniele Gorla, and Daniele Varacca.  
Semantic Subtyping for Objects and Classes.

In *Formal Techniques for Distributed Systems - Joint IFIP WG 6.1 International Conference, FMOODS/FORTE, Held as Part of the 8th International Federated Conference on Distributed Computing Techniques, DisCoTec*, volume 7892 of *LNCS*, pages 66–82. Springer, 2013.

## Component Extension of Core ABS 1/2

$P ::= \overline{DI} \{ s \}$   
 $DI ::= D \mid F \mid I \mid C$   
 $T ::= v \mid D[\langle \overline{T} \rangle] \mid (I, r)$   
 $r ::= \perp \mid G[\overline{f : \overline{T}}] \mid \alpha \mid \mu\alpha.r$   
 $D ::= \mathbf{data} D[\langle \overline{T} \rangle] = \text{Co}[(\overline{T})] \mid \text{Co}[(\overline{T})];$   
 $F ::= \mathbf{def} T \text{ fun}[\langle \overline{T} \rangle](\overline{T} x) = e;$   
 $I ::= \mathbf{interface} I [\mathbf{extends} \overline{I}] \{ \mathbf{port} \overline{T} x; \overline{S} \}$   
 $C ::= \mathbf{class} C[(\overline{T} x)] [\mathbf{implements} \overline{I}] \{ \overline{FI} \overline{M} \}$   
 $FI ::= [\mathbf{port}] T x$   
 $S ::= [\mathbf{critical}] (G, r) T m(\overline{T} x)$   
 $M ::= S \{ s \}$



## Component Extension of Core ABS 1/2

$s ::= \mathbf{skip} \mid s ; s \mid T x \mid x = z \mid \mathbf{await} g$   
 $\mid \mathbf{if} e \mathbf{then} s \mathbf{else} s \mid \mathbf{while} e \{ s \} \mid \mathbf{return} e$   
 $\mid \mathbf{rebind} e.p = z \mid \mathbf{suspend}$

$z ::= e \mid \mathbf{new} [\mathbf{cog}] C(\bar{e}) \mid e.m(\bar{e}) \mid e!m(\bar{e}) \mid \mathbf{get}(e)$

$e ::= v \mid x \mid \mathbf{fun}(\bar{e}) \mid \mathbf{case} e \{ \bar{p} \Rightarrow \bar{e}_p \} \mid \mathbf{Co}[(\bar{e})]$

$v ::= \mathbf{true} \mid \mathbf{false} \mid \mathbf{null} \mid \mathbf{Co}[(\bar{v})]$

$p ::= \_ \mid x \mid \mathbf{null} \mid \mathbf{Co}[(\bar{p})]$

$g ::= e \mid e? \mid \|e\| \mid g \wedge g$

## Standard $\pi$ -types

$\tau ::=$	$\emptyset[\tilde{T}]$	channel with no capability
	$l_i[\tilde{T}]$	linear input
	$l_o[\tilde{T}]$	linear output
	$l_{\#}[\tilde{T}]$	linear connection
$T ::=$	$\tau$	linear channel type
	$\langle l_i.T_i \rangle_{i \in I}$	variant type
	$\#T$	standard channel type
	Bool	boolean type
	...	other constructs

# Session Types

$q ::=$	$\text{lin} \mid \text{un}$	qualifiers
$p ::=$	$!T.U$	send
	$?T.U$	receive
	$\oplus\{l_i : T_i\}_{i \in I}$	select
	$\&\{l_i : T_i\}_{i \in I}$	branch
$T ::=$	$q p$	qualified pretype
	$\text{end}$	termination
	$\text{Bool}$	boolean type

## Encoding of session types

$\llbracket \text{end} \rrbracket$	$\stackrel{\text{def}}{=} \emptyset \ []$	(E-END)
$\llbracket !T.U \rrbracket$	$\stackrel{\text{def}}{=} \ell_o[\llbracket T \rrbracket, \llbracket \bar{U} \rrbracket]$	(E-OUT)
$\llbracket ?T.U \rrbracket$	$\stackrel{\text{def}}{=} \ell_i[\llbracket T \rrbracket, \llbracket U \rrbracket]$	(E-INP)
$\llbracket \oplus\{l_i : T_i\}_{i \in I} \rrbracket$	$\stackrel{\text{def}}{=} \ell_o[\langle l_i - \llbracket \bar{T}_i \rrbracket \rangle_{i \in I}]$	(E-SELECT)
$\llbracket \&\{l_i : T_i\}_{i \in I} \rrbracket$	$\stackrel{\text{def}}{=} \ell_i[\langle l_i - \llbracket T_i \rrbracket \rangle_{i \in I}]$	(E-BRANCH)

## Encoding of session processes

$$\begin{aligned} \llbracket \mathbf{0} \rrbracket_f &\stackrel{\text{def}}{=} \mathbf{0} \\ \llbracket x! \langle v \rangle . P \rrbracket_f &\stackrel{\text{def}}{=} (\nu c) f_x! \langle v, c \rangle . \llbracket P \rrbracket_{f, \{x \mapsto c\}} \\ \llbracket x?(y) . P \rrbracket_f &\stackrel{\text{def}}{=} f_x?(y, c) . \llbracket P \rrbracket_{f, \{x \mapsto c\}} \\ \llbracket x \triangleleft l_j . P \rrbracket_f &\stackrel{\text{def}}{=} (\nu c) f_x! \langle l_j - c \rangle . \llbracket P \rrbracket_{f, \{x \mapsto c\}} \\ \llbracket x \triangleright \{l_i : P_i\}_{i \in I} \rrbracket_f &\stackrel{\text{def}}{=} f_x?(y) . \mathbf{case } y \mathbf{ of } \{l_i - c \triangleright \llbracket P_i \rrbracket_{f, \{x \mapsto c\}}\}_{i \in I} \\ \llbracket \mathbf{if } v \mathbf{ then } P \mathbf{ else } Q \rrbracket_f &\stackrel{\text{def}}{=} \mathbf{if } f_v \mathbf{ then } \llbracket P \rrbracket_f \mathbf{ else } \llbracket Q \rrbracket_f \\ \llbracket P \mid Q \rrbracket_f &\stackrel{\text{def}}{=} \llbracket P \rrbracket_f \mid \llbracket Q \rrbracket_f \\ \llbracket (\nu xy) P \rrbracket_f &\stackrel{\text{def}}{=} (\nu c) \llbracket P \rrbracket_{f, \{x, y \mapsto c\}} \end{aligned}$$

## Subtyping in standard $\pi$ -calculus

$$\frac{}{T \leq T} \text{ (S}\pi\text{-REFL)} \qquad \frac{T \leq T' \quad T' \leq T''}{T \leq T''} \text{ (S}\pi\text{-TRANS)}$$

$$\frac{\tilde{T} \leq \tilde{T}'}{l_i[\tilde{T}] \leq l_i[\tilde{T}']} \text{ (S}\pi\text{-ii)} \qquad \frac{\tilde{T}' \leq \tilde{T}}{l_o[\tilde{T}] \leq l_o[\tilde{T}']} \text{ (S}\pi\text{-oo)}$$

$$\frac{I \subseteq J \quad T_i \leq T'_j \quad \forall i \in I}{\langle l_i - T_i \rangle_{i \in I} \leq \langle l_j - T'_j \rangle_{j \in J}} \text{ (S}\pi\text{-VARIANT)}$$

# Polymorphism

Example of polymorphism in the  $\pi$ -calculus with/without sessions:

$x : !\langle X; D \rangle.\text{end}$  ,  $y : ?\langle X; D \rangle.\text{end}$

$\vdash x!\langle \text{Int}; 5 \rangle \mid y?(z).\text{open } z \text{ as } (X; w) \text{ in } nj!\langle w \rangle$

$\longrightarrow \text{open } \langle \text{Int}; 5 \rangle \text{ as } (X; w) \text{ in } nj!\langle w \rangle$

$\longrightarrow nj!\langle 5 \rangle$

## Semantics of Bounded Polymorphism

$$(\nu xy)(x \triangleleft l_j(B).P \mid y \triangleright \{l_i(X_i <: B_i) : P_i\}_{i \in I} \mid R) \rightarrow$$
$$(\nu xy)(P \mid P_j[B/X_j] \mid R) \quad j \in I$$

$$\mathbf{case} \ l_j(B) \text{-}\nu \ \mathbf{of} \ \{l_i(X_i \leq B_i) \text{-}x_i \triangleright P\}_{i \in I} \rightarrow P_j[B/X_j][\nu/x_j] \quad j \in I$$



## Higher-order constructs

$\sigma ::=$	$T$	general type
	$\diamond$	process type
$T ::=$	$\text{Unit}$	unit type
	$T \rightarrow \sigma$	functional type
	$T \xrightarrow{1} \sigma$	linear functional type
$P ::=$	$PQ$	application
	$v$	values
$v ::=$	$\lambda x : T. P$	abstraction
	$\star$	unit value

## Encoding Higher-Order

$$\llbracket T \xrightarrow{1} \sigma \rrbracket \stackrel{\text{def}}{=} \llbracket T \rrbracket \xrightarrow{1} \sigma$$

$$\llbracket T \rightarrow \sigma \rrbracket \stackrel{\text{def}}{=} \llbracket T \rrbracket \rightarrow \sigma$$

$$\llbracket \lambda x : T. P \rrbracket_f \stackrel{\text{def}}{=} \lambda x : \llbracket T \rrbracket. \llbracket P \rrbracket_f$$

$$\llbracket PQ \rrbracket_f \stackrel{\text{def}}{=} \llbracket P \rrbracket_f \llbracket Q \rrbracket_f$$

Where  $\sigma ::= T \mid \diamond$

## On progress for sessions

### Definition (Progress)

A process  $P$  has *progress* if for all  $\mathcal{C}[\cdot]$  such that  $\mathcal{C}[P]$  is well-typed,  $\mathcal{C}[P] \rightarrow^* \mathcal{E}[R]$  (where  $R$  is an input or an output) implies that there exist  $\mathcal{C}'[\cdot]$ ,  $\mathcal{E}'[\cdot][\cdot]$  and  $R'$  such that  $\mathcal{C}'[\mathcal{E}[R]] \rightarrow^* \mathcal{E}'[R][R']$  and  $R \bowtie_{\{x,y\}} R'$  for some  $x$  and  $y$  such that  $(\nu xy)$  is a restriction in  $\mathcal{C}'[\mathcal{E}[R]]$ .

## Results for Progress

### Theorem (Progress $\Leftrightarrow$ Lock-freedom)

*Let  $P$  be a well-typed closed process. Then  $P$  is livelock-free if and only if  $P$  has progress.*

### Theorem (Progress $\Leftrightarrow$ Closed Lock-Free)

*If  $P$  is well-typed then  $P$  has progress if and only if  $\text{close}(P)$  is livelock-free.*

## Typing Progress

- 1: **procedure** PROGRESS( $\Gamma, P$ )
- 2:     Check  $\Gamma \vdash P$
- 3:     Build  $\text{close}(P)$  from  $\Gamma$
- 4:     Encode  $\llbracket \text{close}(P) \rrbracket_f = P'$
- 5:     **return** TyPiCal( $P'$ )
- 6: **end procedure**